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Title: Pension systems from a tax/labour market perspective

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Abstract:
Most empirical work tends to treat public pensions outlays as being irrelevant in the context of defining marginal incentives to work. This tradition is likely to misrepresent both levels and in particular cross-country variations in effective marginal tax rates. First, public pensions systems within the OECD areas have vastly different levels of ambitions in terms of providing mandatory high replacement rates as well as the redistribution within the pension system. In some countries, higher earnings in working life provide a substantially higher public pension in retirement age while in other countries zero additional pension rights are being accrued. This working paper discusses some of the issues relevant when designing public pension systems seen from a labour market and cross-country perspective

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[1] The views expressed in the working paper are those of the author, not necessarily the Ministry of Finance

Taxes as opposed to deferred benefits in pension systems

A central theme in public economics is the role that taxes play in influencing economic incentives to save, invest and work. A higher marginal tax rate on earnings will for example reduce the net-of-tax benefits from working more hours or undertake studies to boost your future earnings capacity.

Empirical studies require a clarification of what constitutes a tax as opposed to payment for a public service. There is a clear difference between a publicly owned gym club charging a fee as a condition for access to its weightlifting instruments *and* tax authorities requiring a raise of your income tax rate to finance an increase in the foreign aid bill. In the former case, you presumably only pay the fee if the perceived value at least equal the price of the ticket, while in the latter case, the payment is mandatory and your benefits – for example the reduction of number of starving children in Africa – is largely independent of your own marginal contribution.

The OECD[2] has for this purpose outlined some general principles used in the context of its own work in the fiscal field. To be treated as taxes, payments should be:

- Compulsory
- Paid to general government
- Non-requited, ie. do not give rise to an equivalent benefit.

The OECD classification recognises[3] that social security contributions presents a borderline case see *also box 1.* These contributions are in most countries linked to the funding of specific spending programmes such as pensions where historical, individual contributions have a major impact on entitlements. In other words, the requirement of non-requital is not fully respected but social security contributions paid to central government institutions are none-the-less included in the calculation of tax rates as part of the social security contributions.

It also implies that the institutional set up of a pension systems matters for the tax/non-tax classification: a system with compulsory payments to a privately managed pension system will not be treated as a tax while broadly the same system managed by central government may well be included[4]. In practice this may not be a major problem as most compulsory systems with major elements of redistribution, in other words non fully requited payments, will tend to be run by central government agencies while private systems typically are largely based upon individual benefits and depending on the contributors earnings history. This emphasis on the institutional set-up can be more problematic in the future as publicly managed reforms tend to become more "acterial" while governments may at the same time encourage a development of (quasi)compulsory private systems with possible some elements of redistribution.

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[2] See for example in OECD(2004a) page 275.

[3] See for example in OECD(2004a) same page as above and the annex B enclosed on the treatment of social security contributions in OECD revenue statistics.

[4] Compulsory contributions to privately managed systems are though in principle to be counted in the OECD revenue statistics as part of "Financing Social Benefits" which is meant to show compulsory contributions to private-sector institutions that provide social benefits.

Box 1   Borderline cases and consistency across countries:
             Are pension schemes de facto/de jure compulsory and non-requited?

Australia's super-annuation fund: fully mandatory, no major redistribution and not counted as taxes as operated by private fund. As mandatory they are included in OECD's calculation of replacement rates in retirement age. However, they are not included in OECD's *Revenue Statistics* for financing social benefits (part III), possible conflicting with the standard guide.

Iceland's occupational schemes: fully mandatory, no major redistribution not counted as taxes. As they are mandatory, they are included in the OECD's calculation of replacement rates etc. in OECD(2005b). They are also included as a compulsory contribution for private sector benefits in OECD *Revenue Statistics.*

Sweden, Netherlands: private, quasi-mandatory occupational schemes, no major redistribution, not counted as taxes. OECD included them in the pension calculations because they are nearly universal but recognise it is a borderline case. In the Netherlands, there are very large compulsory private contributions in *Revenue Statistics* under the heading *"Financing of social benefits"* in line with the guide but nothing for Sweden.

Canada, UK, Denmark and US: occupational schemes very important, but less coverage than in Sweden and Netherlands. The schemes are not included in pension calculation, not counted as taxes. OECD has precisely because of their wide coverage made supplementary calculations on replacements rates including these schemes. UK reports the mandatory component of private pension schemes while they are not included for Denmark possible conflicting with the guide. Including mandatory, privat pension schemes, coverage in Denmark compares to Sweden and Netherlands.

Denmark: The issue of non-requitance in the context of compulsory payments is illustrated by the treatment of the Special Pension scheme (SP). It was introduced in 1998, requiring that households paid 1 per cent of earned income into a defined contribution pension fund with no redistribution. As a consequence, it was not treated as a tax (compulsory yes, but no redistribution). In 1999, it was changed: every contributor was credited with the same payment for the calculation of future benefits while the financing scheme was unchanged. It was thus changed into a tax (compulsory and with redistribution). In 2001, it was changed back to the original system, moving the system again out of the tax system.

Given the highly different organisation for financing social spending schemes within the OECD area, this is potentially a significant point when comparing marginal and average tax rates. The average rate of social security contributions plus pay-roll taxes as a per cent of GDP was 9½ in 2003, but the variation is substantial *cf. table 1.* In seven countries (Australia, Denmark, Iceland, Mexico, Ireland, Korea and Canada, the share was around 5 per cent of less, while the it exceeded 13 per cent in eight countries (Netherlands, Slovak Republic, Germany, Belgium, Austria, Czech Republic, France and Sweden).

The considerable variation in social security contributions partly reflects more institutional features such as whether to operate with earmarked financing to specific programmes as opposed to use of general taxation which may not necessarily be important in terms of the treatment of social security contributions as taxes or not. In Sweden for example, all employers pay roughly 10 per cent of their

wage bill as a contribution to "retirement pensions" but there is no link between what employers pay in and what the individual employee gets out in pension.

Table 1. The role of social security contributions in OECD countries, 2003

Tax revenue as percentage of GDP

	2000 Social Security	3000 Payroll	**2000+3000**	1000 Income & Profits	4000 Property	5000 Goods & Services	6000 Other	**TOTAL**
NEW ZEALAND	0.0	0.0	**0.0**	20.8	1.8	12.3	0.0	34.9
DENMARK 1)	1.2	0.2	**1.4**	29.0	1.8	16.0	0.2	48.3
AUSTRALIA	0.0	1.8	**1.8**	17.5	3.0	9.4	0.0	31.6
ICELAND	3.4	0.0	**3.4**	17.6	2.4	16.3	0.1	39.8
MEXICO	3.2	0.3	**3.6**	5.0	0.3	10.0	0.1	19.0
IRELAND	4.4	0.2	**4.6**	11.7	1.9	11.4	0.1	29.7
KOREA	5.0	0.1	**5.0**	7.1	3.0	9.4	0.8	25.3
CANADA	5.2	0.7	**5.9**	15.5	3.4	8.8	0.1	33.8
UNITED KINGDOM	6.6	0.0	**6.6**	13.0	4.2	11.6	0.2	35.6
UNITED STATES	6.7	0.0	**6.7**	11.1	3.1	4.6	0.0	25.6
TURKEY	6.8	0.0	**6.8**	7.8	1.0	16.2	0.9	32.8
SWITZERLAND	7.5	0.0	**7.5**	12.6	2.5	6.9	0.0	29.5
JAPAN	9.7	0.0	**9.7**	7.7	2.6	5.1	0.1	25.3
NORWAY	10.0	0.0	**10.0**	18.8	1.1	13.5	0.0	43.4
LUXEMBOURG	11.5	0.0	**11.5**	15.0	3.1	11.6	0.1	41.3
PORTUGAL	11.7	0.0	**11.7**	9.1	1.5	13.6	1.1	37.1
FINLAND	12.0	0.0	**12.0**	17.3	1.1	14.3	0.1	44.8
SLOVAK REPUBLIC	12.3	0.0	**12.3**	6.9	0.6	11.2	0.0	31.1
SPAIN	12.3	0.0	**12.3**	9.8	2.6	9.8	0.3	34.9
ITALY	12.7	0.0	**12.7**	13.3	3.4	11.1	2.6	43.1
HUNGARY	11.7	1.0	**12.7**	9.5	0.8	15.2	0.3	38.5
GREECE	12.9	0.0	**12.9**	8.3	1.6	12.8	0.1	35.7
NETHERLANDS	14.1	0.0	**14.1**	9.9	2.0	12.3	0.5	38.8
POLAND	14.1	0.2	**14.4**	6.2	1.4	12.2	0.0	34.2
GERMANY	14.4	0.0	**14.4**	9.7	0.8	10.4	0.1	35.5
BELGIUM	14.4	0.0	**14.4**	17.7	1.5	11.2	0.6	45.4
CZECH REPUBLIC	16.4	0.0	**16.4**	9.6	0.5	11.2	0.0	37.7
SWEDEN	14.7	2.5	**17.2**	18.3	1.6	13.3	0.2	50.6
AUSTRIA	14.5	2.7	**17.2**	12.8	0.6	12.1	0.4	43.1
FRANCE	16.4	1.1	**17.4**	10.1	3.2	11.1	1.6	43.4
Unweighted average:								
OECD TOTAL	9.5	0.4	**9.9**	12.6	1.9	11.5	0.4	36.3
OECD AMERICA	5.1	0.3	**5.4**	10.6	2.3	7.8	0.1	26.1
OECD PACIFIC	3.7	0.5	**4.1**	13.3	2.6	9.1	0.2	29.3
OECD EUROPE	11.1	0.3	**11.5**	12.8	1.8	12.4	0.4	38.9
EUROPEAN UNION (15)	11.6	0.4	**12.0**	13.7	2.1	12.2	0.5	40.5
EUROPEAN UNION (19)	12.0	0.4	**12.4**	12.5	1.8	12.2	0.4	39.4

Source: OECD Online Revenue Database

1)Denmark has a so-called "Arbejdsmarkedsbidrag" ("Labour market contribution") calculated as 8 per cent of gross earnings which mostly have the character of a general tax and therefore by OECD is classified as an income tax.

Institutional parameters of public pension systems differ significantly

However, the variations reflect also underlying fundamentally different objectives within the various programmes, which is clearly the case for mandated public pension systems. Five countries (New Zealand, Ireland, Canada, Denmark and United Kingdom) have essentially a *flat* level of benefits, producing a highly progressive pattern of replacement rates, i.e. pension income relative to previous earnings *cf. table 2*. For persons with low incomes, replacement rates are thus relatively high – ranging from 63 to 96 per cent for a person earning 50 per cent of an average production worker (APW)) – while these countries at the same time provide the lowest replacement rates at the upper end of the earnings distribution, at or below 30 per cent for a person earning 2½ times APW earnings.

Table 2. Net replacement rates by earnings level, mandatory pension programmes, men
Per cent of individual pre-retirement net earnings

	…….Individual earnings, multiple of average……						2 1/2 relative to 1/2 APW
	0.5	0.75	1	1.5	2	2.5	
New Zealand	77.1	52	39.5	27.9	22	18.1	23%
Canada	89.4	67.6	57.1	39.5	30.6	25.1	28%
Ireland	63	47	36.6	27.4	21.9	18.3	29%
United Kingdom	78.4	57.7	47.6	38.2	29.8	24.7	32%
Denmark	95.6	68	54.1	42.5	35.5	30.8	32%
Czech Republic	88.3	68.3	58.2	42.9	35.3	31	35%
Australia	77	61.2	52.4	43.1	36.5	31.3	41%
Korea	65.3	51.4	44.3	38.1	34	27.8	43%
Belgium	82.7	63.8	63.1	53.3	42.7	36	44%
Japan	80.1	66.3	59.1	51.9	44.3	35.8	45%
Switzerland	71.4	68.9	67.3	53	41.4	34.3	48%
Norway	85.5	73.1	65.1	58.2	50.1	42.8	50%
Iceland	95.8	77.1	65.9	54.1	57.2	55.1	58%
United States	61.4	54.6	51	44.9	39	35.5	58%
France	98	70.8	68.8	62.6	59.2	57	58%
Turkey	113.2	106.7	103.3	99.9	84.3	66.8	59%
Austria	91.2	93.4	93.2	93.5	79.3	63.2	69%
Portugal	115.9	79.8	79.8	84.4	86.3	86.9	75%
Spain	88.7	89.4	88.3	88.4	83.4	68.8	78%
Luxembourg	125	115	109.8	105.6	104.2	100.1	80%
Sweden	90.2	76.4	68.2	70.1	74.3	75	83%
Finland	90.7	78.8	78.8	79.2	78.3	79.3	87%
Mexico	50.4	46.4	45.1	44.3	44.1	44.2	88%
Germany	61.7	66.6	71.8	79.2	67	54.2	88%
Hungary	86.6	90.9	90.5	99.1	92.6	81.8	94%
Italy	89.3	88	88.8	88.4	89.1	89	100%
Greece	99.9	99.9	99.9	99.9	99.9	99.9	100%
Netherlands	82.5	88.2	84.1	85.8	83.8	82.8	100%
Poland	69.6	69.7	69.7	69.8	70.5	71	102%
Slovak Republic	58.2	59.4	60.2	63.1	65.7	67.8	116%
OECD average	**84.1**	**73.2**	**68.7**	**64.3**	**59.4**	**54.5**	**65%**

Source: OECD Pension models, OECD secretariat.

At the other end of the spectrum, five countries (Italy, Greece, Netherlands, Poland and Slovak Republic) operate with mandated pension programmes that have nearly flat replacement *rates* (replacement rates for a person earning 2½ times APW earnings are at least at the level of the replacement rates of the persons with half the earnings of an APW).

The insurance as opposed to the tax/redistribution aspect in a number of countries with high mandatory replacements rates is also illustrated by the degree of progressiveness in social security contributions. Quite a few countries have marginal rates of social contributions that are either zero once the income exceeds 2 times the earnings of an APW (Austria, Germany, Netherlands) or is at least significantly lower than at the bottom end of the wage distribution (France, Sweden, Belgium and UK) *cf table 3*. The counterpart to such lower contribution rates is typically that higher earnings do not lead to higher accruals of benefits as discussed also below.

Table 3. Marginal rate of social contributions and progression in rates

| | ------------------Single – no children------------------ | | | | | --------------------Married – 2 children-------------- | | | | |
	0,5- ,75	0,75 - 1	1 - 1,5	1,5 – 2	2 - 2,5	0,5-,75	0,75 – 1	1 – 1,5	1,5 - 2	2 - 2,5
Austria	32,7	32,7	32,7	28,1	0,0	32,7	32,7	32,7	28,1	0,0
Belgium	45,0	38,0	36,2	36,2	36,0	45,7	36,2	36,1	35,4	35,4
Denmark	8,0	8,0	8,0	8,0	8,0	8,0	8,0	8,0	8,0	8,0
Finland	25,0	25,0	25,0	25,0	25,0	25,0	25,0	25,0	25,0	25,0
France.................	46,8	44,3	37,1	35,1	35,1	46,8	44,3	37,1	35,1	35,1
Germany............	34,2	34,2	28,2	7,0	0,0	34,2	34,2	28,2	7,0	0,0
Greece	34,3	34,3	34,3	34,3	29,7	33,9	33,9	34,1	34,2	30,0
Ireland	25,3	15,1	15,1	12,1	11,8	25,3	15,1	15,1	12,1	11,8
Italy	31,8	31,8	31,8	32,2	32,5	31,8	31,8	31,8	32,2	32,5
Luxembourg	22,7	23,8	23,8	23,8	23,8	22,3	23,2	23,8	23,8	23,8
Netherlands	46,4	32,4	-3,6	0,0	0,0	46,0	32,0	-3,6	0,0	0,0
Portugal..............	28,1	28,1	28,1	28,1	28,1	28,1	28,1	28,1	28,1	28,1
Spain	28,3	28,3	28,3	23,2	0,0	28,3	28,3	28,3	23,2	0,0
Sweden	30,0	29,9	28,1	24,7	24,7	30,0	29,9	28,1	24,7	24,7
United Kingdom.	19,5	19,5	19,5	16,0	10,6	19,5	19,5	19,5	16,0	10,6
United States	14,2	14,2	14,2	14,2	14,2	14,2	14,2	14,2	14,2	14,2

Source: OECD Tax and Benefit Models, own calculations.

The concept of pension wealth

By definition, the divergent structure of replacement rates must reflect the fact that marginal benefit accruals are very different. For the former group of five with flat benefits *levels*, increasing earnings do not imply higher future benefits while this is clearly the case for the latter group of eight countries with relatively flat replacement *rates*,

This can be illustrated by the concept of marginal accrual of pension wealth for different earnings groups[5]. Net pension wealth is defined as the present value of future pension payouts for a person entering retirement as a function of his/her history of earnings. It can be interpreted as the insurance premium required at the age of retirement to pay for the future stream of pension payouts given the expected life expectancy at that age *cf box 2*. The marginal accrual of pension wealth is thus the increase in pension wealth at the age of retirement if total working life earnings are increased to the next multiple of earnings, for example from ½ to ¾ of APW earnings.

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[5] The data are taken completely from OECD(2005a), a short summary of the methodology for estimating pension wealth is contained in annex A.

Box 2   Calculating net pension wealth

The key variable to be calculated is the stream of future benefits that a worker can be expected to receive when the retirement age is reached. This stream of future benefits is then converted back into a present value at the time of retirement using a discount rate. This wealth estimate is then normalised by the earnings of an average production worker in the various countries. To get there the OECD combines national parameters with a number of common macroeconomic assumptions covering all countries.

The national parameters are first of all the prime characteristics of the pension system such as how the stream of historic earnings determines future benefits, whether post-retirement benefits are inflated by growth in earnings or consumer prices, and life expectancy at the time of retirement (the longer you receive a given annual benefit, the larger its total size and hence present value at retirement).

The common assumptions cover real earnings growth (2 per cent per year), consumer price inflation (2½ per cent), general real discount rate for calculating present values (2 per cent), real return on genuinely funded, defined contribution schemes (3½ per cent).

See also chapter 3: "Modelling Pension Entitlements" in OECD(2005b).

For Ireland and New Zealand, that have complete flat rate benefits, ie. the same amount is paid to all irrespective of earlier earnings level, the marginal accrual is by definition zero *cf table 4*. By contrast, increasing work life earnings from for example ½ to 2 times APW earnings will increase net pension wealth at retirement with 3 times or more in thirteen countries.

Table 4. Marginal increase in pension wealth by increasing earnings to next multiple of APW earnings

Multiple of economy-wide average earnings

| | ....Individual earnings.................. | | | | | Top to bottom ratio of |
	0.5	0.75	1	1.5	2	pension wealth, per cent
Ireland	0.00	0.00	0.00	0.00	0.00	100
New Zealand	0.00	0.00	0.00	0.00	0.00	100
Denmark	0.09	0.09	0.19	0.19	0.19	115
Canada	0.48	0.48	0.01	0.00	0.00	117
UK	0.15	0.31	0.86	0.14	0.00	129
Australia	0.49	0.49	0.61	0.25	0.14	135
Belgium	0.00	1.15	0.94	0.00	0.00	138
Czech Republic	0.78	0.73	0.52	0.52	0.52	153
Iceland	0.31	0.33	0.84	1.66	1.66	172
Norway	1.08	0.84	1.50	0.69	0.26	183
Korea	0.84	0.81	1.60	1.24	0.00	189
Switzerland	2.09	1.94	1.15	0.00	0.00	200
Japan	1.24	1.19	2.28	1.13	0.00	207
US	0.94	0.82	1.32	0.66	0.65	216
France	0.00	1.77	2.78	2.64	2.45	231
Austria	2.32	2.07	4.01	1.00	0.00	263
Turkey	1.97	1.97	3.77	1.67	0.00	281
Luxembourg	3.05	2.70	4.95	4.69	3.78	300
Portugal	0.00	2.31	4.92	4.62	4.08	302
Spain	2.43	2.22	4.57	3.39	0.00	309
Finland	1.16	1.68	3.18	2.87	2.84	309
Germany	1.80	1.92	3.83	1.08	0.00	318
Sweden 1)	1.16	1.11	3.35	3.39	2.73	320
Hungary	2.62	2.14	4.26	2.75	1.45	336
Mexico	1.09	1.09	2.18	2.12	2.11	339
Netherlands	2.19	1.96	3.82	2.85	2.81	367
Italy	2.15	1.85	3.34	3.46	3.24	369
Greece	3.15	3.06	5.12	4.40	4.29	418
Poland	1.60	1.60	3.21	3.21	2.90	459
Slovak Republic	1.99	1.99	3.99	3.99	3.99	500

Source: OECD Pension models, OECD secretariat.

1) The OECD calculations for Sweden has been questioned by Swedish authorities who claim that the link between individual earnings and pension wealth is considerable stronger than evidenced by the calculations. This reservation is relevant for all the following tables.

Marginal accrual of pension wealth can be reconstructed as the share of annual earnings over a person's working life that would have been required to pay for the insurance policy at retirement in the following called *annual marginal pension accrual(AMPA)*[6]. Provided contributors perceived the payments as providing specific, secure, future, net-of-tax benefits this share would in effect represent a

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[6] The method is documented in annex b.

marginal negative income tax on wage income[7]. The issue of perceptions is critical and will be discussed below.

To go from gross wealth at the age of pension to annual marginal pension accruals(AMPA), a number of assumptions and calculations are necessary; they are briefly explained *in box 3 and annex B*. However, one very important caveat is to be underlined: "marginal" accrual should be seen for a stylised person starting a working life of 40 years at the age of 25. In reality marginal accruals may have a much more complicated and, often for the future beneficiary of pension payouts, non-transparent relationship with present increases in earnings. This complexity gives rise to the discussions of the efficacy of such systems as discussed further below. One example is that maximum pension rights will often have been obtained before having worked the assumed 40 years, implying that marginal accruals are higher up to for example 35 years while being zero afterwards as discussed in the context of incentives to early retirement[8]

Converting marginal net wealth to AMPA underlines the importance of perceptions. Nine OECD countries have average AMPAs in excess of 6 percentage points[9] *cf. table 5*. At the other end, seven countries have average AMPSs of 0 to 1 percentage points.

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[7] The concept of net marginal rate of social security contributions were developed extensively in early work in the US (Feldstein(1992)) the point precisely that higher earnings not only imply higher income taxes and social security contributions but also higher future benefits.

[8] *OECD(2004c)*

[9] The marginal benefits for the different earnings groups are weighted according to the average wage dispersion within OECD countries, not national wage dispersions. Apart from a data issue: national wage dispersion are not available in the source material or elsewhere, it has the advantage of securing focus on the difference in underlying parameters rather than national differences in earnings structures.

Table 5. Annual marginal negative tax rate equivalents of marginally accrued pension rights, 2002

	calculated for 5 different groups of wage levels, expressed as share of economy wide average earnings………………					
	0.5	**0.75**	**1**	**1.5**	**2**	**Weight average**
Under 5 percent						
Ireland	0%	0%	0%	0%	0%	0%
New Zealand	0%	0%	0%	0%	0%	0%
Denmark	0%	0%	0%	0%	0%	0%
UK	1%	1%	2%	0%	0%	1%
Canada	2%	2%	0%	0%	0%	1%
Australia	2%	2%	1%	0%	0%	1%
Iceland	1%	1%	1%	3%	3%	1%
Belgium	0%	4%	2%	0%	0%	2%
Czech Republic	3%	3%	1%	1%	1%	2%
US	3%	3%	2%	1%	1%	3%
Korea	3%	3%	3%	2%	0%	3%
Norway	4%	3%	3%	1%	0%	3%
France	0%	6%	5%	5%	4%	4%
Mexico	4%	4%	4%	4%	4%	4%
Japan	4%	4%	4%	2%	0%	4%
Above 5 per cent						
Sweden 1)	4%	4%	6%	6%	5%	5%
Switzerland	7%	7%	2%	0%	0%	5%
Finland	4%	6%	6%	5%	5%	5%
Poland	6%	6%	6%	6%	5%	6%
Portugal	0%	8%	9%	8%	7%	6%
Germany	6%	7%	7%	2%	0%	6%
Turkey	7%	7%	7%	3%	0%	6%
Italy	8%	7%	6%	6%	6%	7%
Austria	8%	7%	7%	2%	0%	7%
Netherlands	8%	7%	7%	5%	5%	7%
Slovak Republic	7%	7%	7%	7%	7%	7%
Spain	9%	8%	8%	6%	0%	8%
Hungary	9%	8%	8%	5%	3%	8%
Luxembourg	11%	10%	9%	8%	7%	10%
Greece	11%	11%	9%	8%	8%	10%

Note: In the above calculations of AMPA an active work period of 45 years was assumed. If a shorter active work period was assumed, the present value of pension wealth and AMPA would increase. 1) same reservation as for previous table

Box 3 From pension wealth to annual marginal pension accrual (AMPA)

Essentially, the calculation consists of comparing the present value of future benefits with the present value of future earnings which can be calculated at any point in time. In this, we are considering a hypothetical case with a person just starting his/her working life. The marginal annual pension accrual is thus calculated as the ratio of these two present values.

As real earnings growth is assumed to equal the real discount rate in the OECD calculations and the earnings profile to be flat over time, this implies that the present value of future earnings equal the number of years in employment multiplied by the earnings level.

The present value of pension wealth at the start of the working life equals by definition pension wealth at the start of retirement as described in box 2 discounted by a discounting factor. To calculate the tax rate equivalent of the future benefits, these have to calculated net-of-taxes in retirement (moving from a gross to a net wealth concept). The OECD assumes that the real long term return on funded pension exceeds the standard discount factor as shown in box 2 by 1½ percentage points. One option would thus be to use that higher discount factor to calculate present value at the start of working life rather than the standard discount factor 2 per cent (this is suggested for example in Prescott(2004)).

This discussion gets to the core of some of the present proposals in the US context about "privatising" social security. In a pay-as-you system, real benefits are linked essentially to the real growth rate of the economy which is also in the OECD calculations the value for the standard discount factor (basic premise in a number of macroeconomic models). However, for a number of reasons not to be discussed here be it is possible that long-term, real rates of return of capital markets exceed the growth in productivity. In this case, workers paying implicitly (by way of general taxes) or explicitly (via earmarked social contributions) into a pay-as-you-go system forego the excess rate of return associated with the alternative of a privately funded system.

However, in the calculations in this working paper, pension wealth at retirement has been discounted back to wealth at the start of the working life by using the standard factor of 2 per cent. This is done primarily for one reason. The focus in this paper is on analysing the potential costs to society of having mandatory high replacement rates with limited redistribution between wage earners with different life time wage income financed out of social security contributions or general taxes. So using a "neutral" discount rate of 2 illustrates better, the potential non-redistributive character of a country's pension systems.

If discount rates are higher, then that is just an additional reason for reforming systems: labour market incentives are more even distorted because workers value future benefits lower implying a higher negative effect on hours worked etc. from tax contribution to pay for pensions. Furthermore, here may be also a loss to society because there are real long term positive effects associated with the capital accumulation in pension funds (Feldstein (1992, 2005a and 2005b) has been a long-term advocate of this view).

Marginal pension benefits in tax rate context

The potential importance of AMPAs in terms of ranking countries according to marginal tax rates is illustrated by *cf table 6*. When not factoring in marginal pension benefit, UK have the lowest average marginal tax rates on labour (MTR) within EU15. But if pension accruals are fully included, Greece, Luxembourg, Spain and Portugal have lower rates than UK (MTR minus AMPA).

At the other end, Belgium, Denmark and Sweden[10] keep their top positions, having all relative flat rate pensions. They thus have high marginal tax rates *despite* pension systems with a low level of life time churning of contributions and benefits. However, countries such as Finland, Germany, France, Austria and Italy move substantially "away" from the top rate countries because of relative earnings dependent pension systems.

Table 6. Composite[1) marginal tax rates with and without deduction of accrued pension benefits

	½ - 3/4		3/4 – 1		1 - 1 ½		1½ - 2		2 2½		Average 1)		
	MTR	MTR - AMPA	MTR	MTR - AMPA	MTR	MTR - AMPA	MTR	MTR - AMPA	MTR	MTR - AMPA	MTR	MTR – AMPA	Diffe rence
USA	27,6	24,5	37,2	34,4	40,8	38,6	43,4	42,3	48,8	47,7	36,3	33,8	2,6
GRC	46,0	36,8	48,2	39,3	55,9	48,4	62,3	55,9	60,9	54,6	51,2	42,9	8,3
PTR	45,5	45,5	51,7	45,2	56,8	49,9	58,8	52,3	61,2	55,5	52,2	47,6	4,6
LUX	45,8	37,8	52,7	45,7	61,4	54,9	66,3	60,2	66,3	61,4	54,6	47,6	7,0
ESP	53,7	46,5	54,7	48,1	55,5	48,7	53,9	48,8	46,6	46,6	54,2	47,7	6,5
GBR	49,5	49,0	49,5	48,5	49,5	48,2	54,0	53,8	56,6	56,6	50,0	49,2	0,8
NLD	63,7	57,6	63,5	58,1	47,9	42,6	61,0	57,0	62,6	58,7	58,7	53,3	5,4
IRE	56,1	56,1	48,9	48,9	59,1	59,1	61,8	61,8	61,5	61,5	55,4	55,4	0,0
ITA	57,4	51,1	61,7	56,2	62,6	57,6	66,3	61,2	68,0	63,2	61,2	55,7	5,5
AUT	60,0	53,5	61,8	55,9	63,6	57,9	65,0	63,6	49,5	49,5	61,3	55,8	5,5
FRA	64,7	64,7	65,8	60,8	62,2	58,2	64,0	60,2	66,9	63,4	64,3	61,3	3,1
DEU	65,9	60,6	68,3	62,6	67,1	61,4	60,2	58,6	59,6	59,6	66,4	61,3	5,1
SWE 5)	64,4	61,4	63,8	60,9	68,2	63,8	73,7	69,2	75,4	71,8	66,4	62,9	3,5
FIN	63,8	60,8	67,5	63,1	69,8	65,6	71,1	67,3	74,3	70,5	67,6	63,8	3,9
DNK	62,7	62,5	66,1	65,9	74,7	74,5	75,3	75,0	75,3	75,0	68,6	68,3	0,2
BEL	77,2	77,2	73,6	70,3	75,0	73,6	76,2	76,2	77,6	77,6	75,4	74,0	1,4
Average 2)	56,5	52,8	58,4	54,0	60,6	56,4	63,3	60,2	63,2	60,8	59,0	55,0	4,0
EU-15 3)	59,3	55,5	60,9	56,2	60,3	55,8	61,2	58,3	61,3	59,5	60,3	56,2	4,1

1)   Weighted by the various fractiles of the earnings distribution
2)   Non-weighted
3)   Weighted by share of work force
4)   The marginal tax rate MTR includes taxes on consumption following the concept suggested in David et al(2002)
5)   Same reservation for Sweden as for previous tables.
Source: OECD (2004a, 2005a, 2005b) and own calculations.   )

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[10] See though footnote to table 4

Are marginally accrued pension benefits negative taxes?

The fact that one can construct AMPA does not imply that they can be equated with negative tax rates for several reasons. The first is that the *simplification of assumptions* necessary for the construction of average AMPAs implies that marginal benefits may in a number of cases be below or above the benchmark cases. To give one example: In some pension systems benefits are based upon a subset of best earnings years. An isolated higher bonus for example early on in the working life where salaries typically are low may thus not be large enough to affect pension benefits – that year's salary will still be below the lowest of so far best years – but may well imply higher social security (or other tax) contributions. Furthermore, the *transparency* of the link between higher earnings and future benefits may be weak for the individual contributor. Hence if the higher social security contribution and/or higher marginal tax rates *is perceived* to be without partial counterpart in terms of higher benefits, then the behaviour of the contributor will not be affected by the actually accrued benefits and only the formal rates are relevant. Finally, *future* benefits are conditional upon the present rules for accruals to be maintained at their present level of generousity. In the present situation of major pressures on pension systems in the context of ageing and non-sustainable long-term positions of public finances in a number of OECD countries, that is strong assumption to make. A second factor that may reduce the value of future benefits is declining working hours for future generations of workers. PAYG systems typically link benefits with development in total yearly, not hourly, earnings: so if present workers believe future workers will work less, then the expected net present values of future pension should be lower than if projected on the basis of current level of hours worked per employee.

However, there is on the other hand no basis for presuming that contributors perceive contributions and taxes to finance mandatory pension systems providing high replacement rates for themselves in retirement age as being "pure" taxes. This conclusion can be reached in a somewhat roundabout fashion. The theoretical and empirical literature strongly suggest boosting mandatory pension programmes and even more private voluntary (often tax subsidied) institutionalised private pension schemes will largely squece out other private savings rather than led to net increases in savings, this implies in particular to medium and higher incomes. In countries with low mandatory replacement rates for medium and high income earners such as UK, US and Denmark, private pension plans prosper precisely because such higher income earners do recognise the need to supplement relative meagre pension benefits with own savings *cf. table 7*. By contrast, countries that have historically relied on publicly financed pension systems with high mandatory formal replacements have typically had very low levels of private institutionalised pension funds.

Table 7. Public pension parameters reflected in size of private pension funds

	Private pension funds	Mandatory public systems		
		Average wealth of pension		
	Share of GDP	Benefits at retirement		Distribution of payouts (Gini-koefficient)
	2003, 00	Men	Women	
Korea...	1,6	6,5	7,6	11.7
Czech Republic..............................	2,5	6,9	8,1	7.8
Italy	2,5	11,1	13,1	26.1
Belgium	3,5	6,5	7,5	9.5
Germany..	3,6	7,7	9,2	20.9
Austria...	4,3	11,0	13,3	21.5
Norway..	4,6	7,7	9,0	14.8
Hungary	5,3	11,7	14,4	25.6
France	5,4	9,5	10,9	14.6
Poland	5,5	7,7	8,1	25.7
Mexico...	5,7	4,7	4,5	23.4
New Zealand	5,7	5,7	6,5	0.0
Spain	6,5	11,3	13,2	23.6
Sweden 1)......................................	7,8	10,9	12,0	21.3
Finland	8,3	11,2	13,3	25.3
Slovak Republic	9,4	7,9	9,6	26.3
Portugal	12,4	10,8	12,6	18.7
Japan	13,1	7,9	8,9	14.2
Denmark	105 [2]	7,3	8,4	2.3
Canada	52,0	6,1	7,1	3.7
Australia	56,1	6,6	7,7	6.8
United Kingdom............................	65,7	5,5	6,3	8.3
United States	66,0	5,2	6,1	16.1
Netherlands	94,1	6,0	6,0	3
Iceland...	102,0	8,6	9,7	10.6
Switzerland	125,5	8,7	10,7	15.2
Total OECD.................................	**61,2**	**8,7**	**10,2**	**16.4**

1) Same reservation as for previous tables

2) 2005 estimate from Danish Ministry of Finance, includes all private pension funds, though with ATP funds excluded.

Source: OECD(2005b and 2005c)

So the likely conclusion is that labour market participants on the whole understands the basic parameters in countries where public pension benefits are relatively strongly linked to historic earnings otherwise they would save more for retirement, but the link is not perceived as a one-to-one relationship: one theoretical additional unit of present net value of pension benefits does not have full negative tax value.

Overall conclusions

The overall conclusion then is probably the following. The disregard of the role of marginal pension benefits in a number of cross-country studies[11] linking marginal tax rates to labour market performance is problematic. Without any attempt of being scientific box 4 suggest that differences within EU-15 and the US are better explained by deducting marginal pension benefits from formal tax rates rather than not doing it.

On the other hand, there are probably significant inefficiencies in public pension systems that aim for high mandatory replacements rates particular in the context of already high marginal tax rates. The key question to be asked is: which social policy objectives are achieved by enrolling medium and high income earners into relative ambitious pension programmes where the degree of redistribution may be limited but whose financing costs are seen none-the-less by contributors to a large extent as taxes and not just mandatory private savings. All EU-15 countries have composite marginal tax rates on labour at 50 per cent or above *cf table 8.* A substantial number of these countries have at the same time pension systems that may produce a substantial amount of "social" deadweight loss in terms of a missed opportunity of lower formal tax rates (column 2). These could be reduced by lowering the life-cycle "churning" of contributions from middle and high income earners to the same groups in retirement. Such reform would lower the average level of pension benefits (column 3) by reducing top replacement rates (leading to lower Gini-koefficents of pensions payouts) and at the same time tax/contribution rates for these earning groups keeping the life-cycle distribution of income between different earning groups roughly unchanged.

Table 8. Ranking of countries in terms of potential gains for reforms

| Formal marginal tax rates		De facto reduction in rates		Pension wealth		Life time churning in pension systems (illustrated by Gini-koefficient)	
Belgium	75	Luxembourg...	12	Luxembourg	17,8	Italy	26,1
Denmark	69	Greece............	11	Greece	12,4	Greece...................	26,0
Finland	68	Spain..............	9	Spain	11,3	Portugal	25,7
Sweden	66	Austria	8	Finland	11,2	Netherlands	25,6
Germany.............	66	Italy	8	Austria	11,1	Finland	25,3
France................	64	Finland	6	Italy	11,1	Spain.....................	23,6
Austria	61	Germany........	5	Portugal	10,8	Luxembourg...........	22,5
Italy	61	Portugal	5	France.................	9,5	Austria	21,5
Netherlands	59	France............	4	Sweden	8,8	Sweden	21,3
Ireland	55	US	3	Germany..............	7,7	Germany...............	20,9
Luxembourg	55	Belgium	2	Denmark	6,7	United States.........	16,1
Spain	54	Sweden...........	1	Belgium	6,5	France...................	14,6
Portugal..............	52	UK.................	1	United Kingdom..	5,5	Belgium	9,5
Greece	51	Denmark........	0	Ireland	5,4	United Kingdom...	8,3
United Kingdom.	50	Ireland	0	United States	5,2	Denmark...............	0,0
United States	36	Netherlands....	0	Netherlands	3,0	Ireland	0,0

Source: same as pre-ceding tables.

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[11] Examples included Prescott(2004), Alesina et al(2005) and David(2004). Prescot acknowledges the theoretical argument of marginal benefits but downplays its practical importance given the relative low lewel of marginal pension pensions in the US. The point here is though that they are most more significant for the countries that he is benchmarking US against.

Less radical reforms of public pensions would attempt to strengthen the actuarial link between life long earnings and benefits both in effect and in presentation. Design features and credibility are crucial. Two systems that confer broadly the same ultimate benefit to a group of contributors with a given life long earnings level may have very different effects on incentives. In the one end, you may have a pay-as-you-go system with complicated rules linking final benefits to perhaps five best earnings years that. At the other end are compulsory systems with clear and transparent links between earnings and benefits with for example annual presentations of earned pension points for individual contributors without necessarily being less ambitious in reaching well defined social objectives including redistribution. Provided overall sustainability of public finances were on track, workers would in this case also have larger confidence in their pension wealth being actually pay-out to them in retirement. Elements of such reforms are already under way in a number of European countries simply due to the pressure ageing puts on pay-as-you pensions systems as documented in a range of surveys.

The point in this paper is that labour market concerns point strongly in the same direction of reform. Indeed, the potential benefits for reform in Europe should dwarf the benefits to be reaped by similar reforms being proposed presently in the US simply because distortions from tax rates rise with their level (and US has markedly lower rates) and because the US public pension system is much less ambitious in terms of pension levels and replacement rates for middle and higher incomes[12].

Box 4: Hours worked: which tax rates explain best?

Marginal tax rates with and with deduction of pension accruals and hours worked
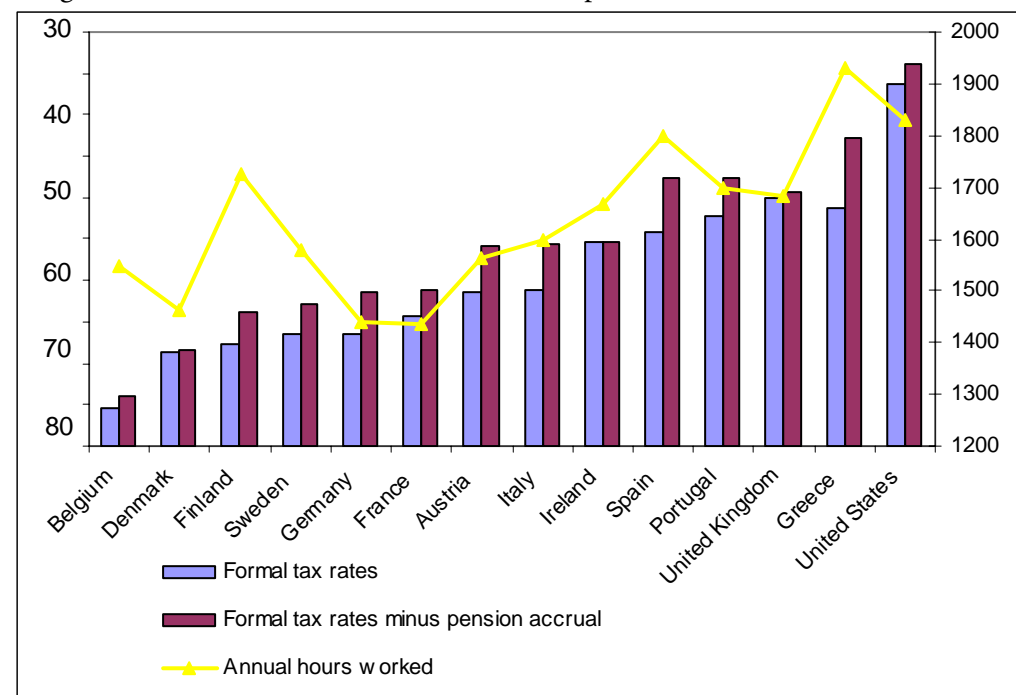
Sources: Average hours worked is taken from Employment Outlook 2005. Tax and pension incentives as calculated in this working paper.

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[12] Martin Feldstein has produced a number of reports outlining the benefits in the US for privatising social security seen from both a labour market and capital market perspective.

Workers in countries with low marginal tax rates (US, GRC and ESP) do tend to put in more hours than workers characterised by high marginal tax rates (SWE, DK and BEL).

However, the central question in this paper is whether pension accruals are well perceived as marginal benefits that also at the margin increases the return on working more hours. This would in practice imply – all other things equal – that tax rates minus pension accruals would be a better predictor of annual hours worked than the pure tax rates. Some support for that can be observed at the top end: clearly GRC, ESP and PRT having the most industrious workers also stand out as having higher relative marginal work incentives when accruals are included. The hard-working Fins are also better explained once pension accruals are included.

Considerable caution should be taken in too firm interpretation of such simple relationships. Working hours are affected by many other factors than marginal tax rates, including incentives embedded in social security systems, and the family tax models used for these calculations do not capture accurately work incentives within the family (e.g the effects on labour market participation of secondary wage earners, incidence on part time work).

In fact, Netherlands was taken out of the graph simply because part-time work (35 per cent of the working population) is significantly higher than in any other OECD country, drawing down significantly the number of average hours work for all employed workers. The high number of hours worked in Finland – for example compared to other Nordic countries with high marginal tax rates – is most certainly also a consequence of lower incidence of part time work (11 per cent against 13 per cent in Sweden, 17 per cent in Denmark and 17 per cent for EU15). Finally, the measurement of numbers of working hours is quite fickle and probably becoming increasingly so as more and more persons are no longer paid by the hour, requiring more sophisticated surveys to establish number of hours worked (in the US the proportion of person paid by the hour has fallen from 62 to 58 per cent of the work force over the last 10 years (Lemieux (2003)).

Finally, as stated in the main body of the text, there is no reason to expect that such average, model based comparisons of marginal benefit accruals provide the same marginal benefits either de facto or as perceived in different countries. Consequently, the same measured pension accrual could have quite different labour market effects in different countries.

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Annex A Extract from *OECD Classification of Taxes and Interpretative Guide*
(published in OECD Revenue Statistics)

35. Classified here are all compulsory payments that confer an entitlement to receive a (contingent) future social benefit. Such payments are usually earmarked to finance social benefits and are often paid to institutions of general government that provide such benefits. However, such earmarking is not part of the definition of social security contributions and is not required for a tax to be classified here. However, conferment of an entitlement is required for a tax to be classified under this heading. So, levies on income or payroll that are earmarked for social security funds but do not confer an entitlement to benefit are excluded from this heading and shown under personal income taxes (1100) or taxes on payroll and workforce (3000). Taxes on other bases, such as goods and services, which are earmarked for social security benefits are not shown here but are classified according to their respective bases because they generally confer no entitlement to social security benefits.

36. Contributions for the following types of social security benefits would, *inter alia*, be included: unemployment insurance benefits and supplements, accident, injury and sickness benefits, old-age, disability and survivors' pensions, family allowances, reimbursements for medical and hospital expenses or provision of hospital or medical services. Contributions may be levied on both employees and employers.

37. Contributions may be based on earnings or payroll ("on a payroll basis") or on net income after deductions and exemptions for personal circumstances ("on an income tax basis"), and the revenues from the two bases should be separately identified if possible. However, where contributions to a general social security scheme are on a payroll basis, but the contributions of particular groups (such as the self-employed) cannot be assessed on this basis and net income is used as a proxy for gross earnings, the receipts may still be classified as being on a payroll basis. In principle, this heading excludes voluntary contributions paid to social security schemes. When separately identifiable these are shown in the memorandum item on the financing of social security benefits. In practice, however, they cannot always be separately identified from compulsory contributions, in which case they are included in this heading.

38. Contributions to social insurance schemes which are not institutions of general government and to other types of insurance schemes, provident funds, pension funds, friendly societies or other saving schemes are not considered as social security contributions. Provident funds are arrangements under which the contributions of each employee and of the corresponding employer on his/her behalf are kept in a separate account earning interest and withdrawable under specific circumstances. Pension funds are separately organised schemes negotiated between employees and employers and carry provisions for different contributions and benefits, sometimes more directly tied to salary levels and length of service than under social security schemes. When contributions to these schemes are compulsory or quasicompulsory (*e.g.*, by virtue of agreement with professional and union organisations) they are shown in the memorandum item (refer Section III.B of the Report).

39. Contributions by government employees and by governments in respect of their employees, to social security schemes classified within general government are included in this heading. Contributions to separate schemes for government employees, which can be regarded as replacing general social security schemes, are also regarded as taxes.15 Where, however, a separate scheme is not seen as replacing a general scheme and has been negotiated between the government, in its role as an employer, and its employees, it is not regarded as social security and contributions to it are not regarded as taxes, even though the scheme may have been established by legislation.

40. This heading excludes "imputed" contributions, which correspond to social benefits paid directly by employers to their employees or former employees or to their representatives (*e.g.*, when employers are legally obliged to pay sickness benefits for a certain period).

41. Contributions are divided into those of employees (2100), employers (2200), and selfemployed or non-employed (2300), and then further sub-divided according to the basis on which they are levied. Employees are defined for this purpose as all persons engaged in activities of business units, government bodies, private non-profit institutions, or other paid employment, except the proprietors and their unpaid family members in the case of unincorporated businesses. Members of the armed forces are included, irrespective of the duration and type of their service, if they contribute to social security schemes. The contributions of employers are defined as their payments on account of their employees to social security schemes. Where employees or employers are required to continue the payment of social security contributions when the employee becomes unemployed these contributions, data permitting, are shown in 2100 and 2200 respectively. Accordingly, the sub-heading 2300 is confined to contributions paid by the self-employed and by those outside of the labour force (*e.g.*, disabled or retired individuals).

ANNEX B:

From pension wealth to annual marginal pension accrual (AMPA)

What we want to measure is the required additional yearly payments calculated as a share of annual earnings that would have been necessary over a persons working life to provide a person with the estimated increases in pension wealth estimated by the OECD in Pensions at a glance. The basic approach to estimate these AMPA is explained in box 3 in the main text., the annex attempts to clarify the actual calculations:

First we define the basic variables:

r = the real interest rate used for actuarial discounting real flows and stocks back and forth in time

γ = the yearly growth rate of real earnings,

Pension wealth at the time of retirement is called K and is given for the different multiples of average earnings, with the grouping named with index i and defined it self as a multiple of the earnings of an average production worker. Pension wealth for a person with earnings 1,5 times of APW is therefore named $K_{1,5}$.

The next step is defining the marginal increase in pension wealth when moving to the next multiple of APW earnings which is used in the paper. It is defined as $MPW_i = K_i - K_{i-1}$, and measures the extra pension wealth when going into the next multiple of average earning.

T = The working life, defined as the difference between the retirement age and the start of the working life and keeping in with the

Since looking at MPW, which is expressed as a marginal increase, AMPA also evaluates the share of additional earnings – dW – that would need to be saved.

The stream of pension payments can then be defined as.

$$dW_1 * AMPA + dW_2 * AMPA + dW_3 * AMPA + ... + dW_t * AMPA$$

Every year an amount is saved for pension. The marginal pension wealth is then

$$dW_1 * AMPA * (1+r)^t + dW_2 * AMPA * (1+r)^{t-1} + ... + dW_t * AMPA * (1+r) = dK \quad \text{=MPV}$$

Interest is applied to the payments, so the payments that have been made in the past will be subject to accumulated interest.It is preferred to have the pension wealth in terms of the starting year. Therefore all the terms are discounted back to the starting period.

$$dW_1 * AMPA + \frac{dW_2 * AMPA}{1+r} + \frac{dW_3 * AMPA}{(1+r)^2} + ... + \frac{dW_t * AMPA}{(1+r)^{t-1}} = \frac{dK}{(1+r)^t}$$

dW_1 is the salary in the first year and in the main text the wage is assumed to grow by specific rate every year, therefore $dW_1 = dW_2 * (1+\gamma)$ and onwards $dW_3 = dW_1 * (1+\gamma)^2$.

$$dW_1 * AMPA + \frac{dW_1 * (1+\gamma) * AMPA}{1+r} + \frac{dW_1 (1+\gamma)^2 AMPA}{(1+r)^2} + ... + \frac{dW_1 (1+\gamma)^t * AMPA}{(1+r)^{t-1}} = \frac{dK}{(1+r)^t}$$

Isolating AMPA:

$$dW_1 * AMPA + \frac{dW_1 * (1+\gamma) * AMPA}{1+r} + \frac{dW_1(1+\gamma)^2 AMPA}{(1+r)^2} + ... + \frac{dW_1(1+\gamma)^t * AMPA}{(1+r)^{t-1}} = \frac{dK}{(1+r)^t} \Rightarrow$$

$$AMPA * dW_1 * \left(1 + \frac{1+\gamma}{1+r} + \left(\frac{1+\gamma}{1+r}\right)^2 + ... + \left(\frac{1+\gamma}{1+r}\right)^{t-1}\right) = \frac{dK}{(1+r)^t} \Rightarrow$$

$$AMPA = \frac{dK}{(1+r)^t * dW_1 * \left(1 + \frac{1+\gamma}{1+r} + \left(\frac{1+\gamma}{1+r}\right)^2 + ... + \left(\frac{1+\gamma}{1+r}\right)^{t-1}\right)} = \frac{MPW}{(1+r)^t * \left(1 + \frac{1+\gamma}{1+r} + \left(\frac{1+\gamma}{1+r}\right)^2 + ... + \left(\frac{1+\gamma}{1+r}\right)^{t-1}\right)}$$

In the example in the main text it is assumed that the discount rate and the earnings growth rate are the same. This implies that the above formulae becomes far more simply:

$$AMPA = \frac{MPW}{(1+r)^t * (1 + 1 + 1^2 + ... + 1^{t-1})} = \frac{MPW}{(1+r)^t * T}$$

The intuition behind this result is simple. If the growth rates are the same then the net present value of contributions is the same irrespective of the timing. For example: the value of the first years contribution in the second year is equal to first years real earnings multiplied by the contribution rate plus accumulated interest. The value of the second years contribution in the second year is equal to the contribution rate multiplied by the earnings in the second years. However, as the order of factors are irrelevant and the first years accumulated interest is being multiplied by the same factor as second years earnings, the value must the same. If the real interest rate had been zero (and real earnings growth also), then the contribution rate would have been equal to MPW – the real additional pension wealth at retirement – divided by number of contribution years. This would follow from the fact that the real value of all years contribution would be equal, no earnings growth and no real interest. However, with positive real interests, the required contribution rate is lower which is the intuition behind the $(1+ r)^2$ term.

In the case that the two rates are not alike, it is a problem described by mathematicians as a finite geometrical sum.[13]

For understanding what is done let us look at an example with a person worker for 4 years with 3 different scenarios regarding interest and earnings growth and how it affects the replacement rate. In this example there is a flat savings of 10 %.

Table A2.1

	A	B	C
Discount rate	1,02	1,02	1,04
Real earnings growth rate	1,02	1,04	1,02
Excess earning growth rate	1,00	1,02	0,98
Yearly savings	0,10	0,10	0,10

In the example case A is the one used in the paper were discount rate and earnings growth equal. In B the earnings growth 2 percentage points more, in C the other way around. Every year 10 % of salary is saving for pension. In the next table the result of this is shown.

Table A2.2

A	0	1	2	3	K as ratio of wage

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[13] See Berck and Sydsæter, Matematisk Formelsamling p 31 or any other standard mathematical economics book.

Salary	10	10,20	10,40	10,61	
Pension payments	1,00	1,02	1,04	1,06	
Pension wealth	1,00	2,04	3,12	4,24	0,4

B	0	1	2	3	K as ratio of wage
Salary	10	10,40	10,82	11,25	
Pension payments	1,00	1,04	1,08	1,12	
Pension wealth	1,00	2,06	3,18	4,37	0,39

C	0	1	2	3	K as ratio of wage
Salary	10	10,20	10,40	10,61	
Pension payments	1,00	1,02	1,04	1,06	
Pension wealth	1,00	2,06	3,18	4,37	0,41

Case B: The interest cannot keep up with the earnings growth and because of the flat savings rate in the end the marginal replacement rate defined as wealth over final earnings is lower.

Case C: This time it is the other way around. The interest outperforms the earnings growth and therefore the early savings weigh more meaning that it is not necessary to save as much to achieve the same replacement rate as in A.

On the other hand if the pension wealth was known which is the case in the main text the above formulas can be used to calculate AMPA and therefore how much is needed to save every year to achieve the wanted replacement rate.

Table A2.3

A	0	1	2	3
Salary	10	10,20	10,40	10,61
Pension payments	1,25	1,28	1,30	1,33
Pension wealth	1,25	2,55	3,90	5,31

MPW	5,31		AMPA	0,13

The aim is for MPW to be 50 % of final year earnings. Then AMPA is calculated with the above formulas.

$$AMPA = \frac{5,31}{1,02^3 * 4 * 10} = 0,13$$

0,13 percent of the wage is saved every year and the pension wealth grows so that the pension wealth end at the required target.