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Title: Long-Term Fiscal Indicators: "Sustainability versus Terminal Debt Constraints".[1]

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Abstract:

The paper provides an assessment of the conceptual and quantitative differences between three alternative ways of deriving long-term indicators of fiscal stance, two of which involve truncation of the time horizon while the third one is based on the government's intertemporal constraint only and thus involves an effectiely infinite horizon. It turns out that, for the OECD countries, the two finite-horizon alternatives tend to imply too little fiscal consolidation compared to what is required for "true" fiscal sustainability. In other words, the imposition of a fixed horizon typically leads to an excessively optimistic picture of current fiscal stance and may furthermore complicate cross-country comparison.

[1] The views and analyses presented in the working paper are the sole responsibility of the authors. The papers may therefore include views, which are not necessarily shared by the Ministry of Finance.

1. Alternative approaches to long-term fiscal targets[2]

In the literature and among practitioners a number of alternative ways of deriving forward-looking constraints on current fiscal policy have been proposed. The key problem that the various contributions seek to overcome is how to deal with the infinite horizon of the government's intertemporal budget constraint.

The differences thus relate to the technical assumptions that are invoked in order to derive an index of fiscal stance in a long-term perspective that lends itself to numerical application. The purpose of this paper is to clarify and examine the nature of these differences and assess how they influence the quantitative assessment of fiscal sustainability.

To set the stage, consider the (continuous time) equation of motion for government debt,

$$(1) \qquad \dot{b}_s = (i-\gamma)b_s - p_s$$

where p_s is the time s primary surplus, b_s is government debt, while i denotes the nominal rate of interest and γ the growth rate of nominal GDP. We assume that the rates of growth and interest remain constant through time.

We wish to derive and compare appropriate expressions for the required primary surplus at time $t<s$. Integration of both sides of equation (1) from t to T produces

$$(2) \qquad b_T = b_t e^{(i-\gamma)(T-t)} - \int_{s=t}^{T} p_s e^{(i-\gamma)(T-s)} ds$$

Rearranging terms, and letting T go to infinity, then yields

$$(3) \qquad \int_{s=t}^{\infty} p_s e^{-(i-\gamma)(s-t)} ds = b_t - \lim_{T\to\infty}\left\{ b_T e^{-(i-\gamma)(T-t)} \right\}$$

By imposing the assumption that the limit term is equal to zero, we obtain from equation (3) the familiar intertemporal solvency condition implying that the net present value of future primary surpluses – and hence the excess of tax receipts over non-interest government spending – must equal initial net public debt b_t.

Using equations (2) and (3) we now consider the required primary surplus at time t given three different ways of rendering the assessment of fiscal stance computationally feasible. In the first two approaches, a finite horizon is imposed through the requirement that terminal government debt, i.e. b_T, be equal to some pre-specified value.

We first consider the "standard" approach building on the requirement that terminal debt b_T be equal to initial debt b_t. We call this the "Unchanged Terminal Debt", or UTD, rule. Next, we examine what

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happens when the terminal debt level is obtained by assuming that, in the period from t to T, government budget balance holds on average. This assumption is termed the "Close To Balance", or CTB, rule.

Finally, we consider the consequences of retaining the infinite horizon while imposing constraints on the time path of the primary surplus in order to support computational feasibility. One way of doing this is to assume that, from period T onwards , the primary surplus is constant and equal to its value in period T. An alternative way, which we will use below, is to impose a particular functional form on the time path of the primary surplus. Specifically, if we invoke the assumption of an exponential adjustment path, the integral on the left-hand side of equation (3) may be solved analytically. We call this approach the "Sustainability", or S, rule.

Using these three alternatives we obtain the following expressions for the required primary surplus at time t:

"Unchanged terminal debt (UTD)" [Blanchard *et. al* (1990)]
Setting terminal debt equal to initial debt in equation (2) and solving out yields

$$(4) \qquad p_t^{UTD} = (i-\gamma)b_t + \frac{i-\gamma}{1-e^{-(i-\gamma)(T-t)}} \int_{s=t}^{T} \Delta p_s e^{-(i-\gamma)(s-t)} ds$$

"Close to balance rule (CTB)" [EPC and EU Commission (2003)]
In this case, terminal debt b_T is equal to initial debt b_t divided by the compound nominal income growth factor from period t to T. Inserting in equation (2) and rearranging gives

$$(5) \qquad p_t^{CTB} = \left\{ \frac{1-e^{-i(T-t)}}{1-e^{-(i-\gamma)(T-t)}} \right\} (i-\gamma)b_t + \frac{i-\gamma}{1-e^{-(i-\gamma)(T-t)}} \int_{s=t}^{T} \Delta p_s e^{-(i-\gamma)(s-t)} ds$$

"Sustainability (S)" [Frederiksen 2001a,b]
This approach builds on the requirement that tax rates and per capita government spending – i.e., the fiscal instruments – are maintained indefinitely. In this case, fiscal policy is "sustainable" because no future changes in fiscal instruments are required. Using equation (3) we thus find

$$(6) \qquad p_t^{S} = (i-\gamma)\left\{ b_t + \int_{s=t}^{\infty} \Delta p_s e^{-(i-\gamma)(s-t)} ds \right\}$$

Notice how equation (6) states that the time t required primary surplus is equal to the annuity value of total government (i.e., explicit *plus* implicit) debt, where implicit debt is defined as the present value of the future change in net government spending embodied in current tax and expenditure policies.[3]

Comparison of the three alternative approaches

[3] That is, implicit debt is related to the *additional* future net government spending generated by the current settings of fiscal policy instruments. Alternatively, one might define implicit debt as the present value of the *total* net liabilities of the government vis-à-vis currently living generations, but such a definition is inappropriate given the, essentially, infinite horizon (macro-) approach adopted in this paper and the focus on the necessary adjustment of fiscal policy at time t in order to satisfy the relevant terminal constraint on government debt or, under the S approach, government solvency.

The remainder of the paper is devoted to an assessment of the conceptual as well as quantitative differences between the measures of fiscal stance derived above. As will become evident, the S rule possesses certain key characteristics that make it an attractive benchmark against which to assess the bias caused by imposing a finite horizon under the UTD and CTB rules.

Clearly, as equations (4) through (6) reveal, when T goes to infinity, the UTD and CTB rules converge to the S rule. Hence, in this sense they are identical in the limit. But that limit may be so distant that the justification for using either alternative to the S rule is unfounded. Below, we provide estimates of the quantitative extent of the approximation error.

Before doing so, however, it is useful to focus on certain conceptual differences between the UTD, CTB and S approaches in terms of the way fiscal policy is (explicitly or implicitly) assumed to evolve over time. Under the S approach, initial fiscal instrument settings may be maintained indefinitely. Hence, no changes in tax rates or expenditure standards, even in the extremely distant future, are necessary.

In contrast, equations (4) and (5) show that present value budget balance holds only up until the terminal period under the UTD and CTB methods. Accordingly, because the evolution of the primary surplus after the terminal period is effectively ignored, sooner or later fiscal policy will have to be adjusted in order to ensure that the intertemporal budget constraint of the government is satisfied.

Furthermore, while the UTD method calculates the adjustment of fiscal policy that must be undertaken immediately at time t and maintained until time T, strict adherence to the CTB rule would imply gradual changes in fiscal policy in order to maintain budget balance. However, the magnitude of the fiscal indicator in equation (5) effectively assumes a one-off adjustment that is carried out at time t and maintained until time T. Accordingly, the operational content of the CTB rule in terms of the implied policy prescription remains somewhat unclear.

As emphasized above, the concept of sustainability under the S rule is directly related to fiscal policy instruments. In contrast to the UTD and CTB rules, and because of the infinite horizon, the S approach – by construction – requires **no** subsequent change in fiscal policy.

A useful way of restating this property is to note that under a fiscal policy satisfying equation (6), the "discretionary component" (as distinguished from the "autonomous component" – i.e., the change in the primary surplus due to ageing and other factors) remains constant over time. The discretionary component is simply equal to the initial, required primary surplus in equation (6).

The S approach thus implies that total government net debt service is constant and equal to the annuity value of government net liabilities at the outset. The mirror image of this result is that under a sustainable fiscal policy, total government debt likewise remains constant over time.[4]

In the typical ageing scenario, the primary balance declines gradually during the transition. Therefore, under a fiscal policy satisfying the S rule, *explicit* government debt is reduced in order to offset the increase in *implicit* government debt that occurs as the passage of time converts the future liabilities embodied in current tax rates and expenditure standards into current liabilities with correspondingly higher net present value.

[4] This property is pointed out by Buiter (1985). Strictly speaking, constancy of total debt holds only when the growth-adjusted rate of return is constant.

No similarly attractive interpretation of the evolution of government debt applies under the UTD and CTB approaches. And, as noted above, the discretionary component of the primary surplus will – sooner or later – have to adjust in order to meet the government solvency constraint. In the final part of the paper, we present quantitative estimates of the magnitude of the required (additional) change in discretionary fiscal policy.

In order to quantify the bias implied by the UTD approach, consider the amount by which the sustainable primary surplus exceeds p^{UTD},

$$(7) \qquad p_t^S - p_t^{UTD} = (i-\gamma)\int_{s=T}^{\infty} \Delta p_s e^{-(i-\gamma)(s-t)} ds - \frac{e^{-(i-\gamma)(T-t)}}{1-e^{-(i-\gamma)(T-t)}}(i-\gamma)\int_{s=t}^{T} \Delta p_s e^{-(i-\gamma)(s-t)} ds$$

A positive value of the expression on the right-hand side of equation (7) indicates that the primary surplus given by the UTD method falls short of what is required for fiscal sustainability. The shortfall, in turn, reflects the fact that, by ignoring implicit debt after time T, the UTD approach underestimates the fiscal burden of ageing. On the other hand, the finite horizon assumption gives too much weight to implicit debt obligations in the interim period, i.e. from t to T.

Similarly, the bias under the CTB rule may be expressed as

$$(8) \qquad p_t^S - p_t^{CTB} = \left\{ \frac{e^{-i(T-t)} - e^{-(i-\gamma)(T-t)}}{1-e^{-(i-\gamma)(T-t)}} \right\}(i-\gamma)b_t + \left\{ p_t^S - p_t^{UTD} \right\}$$

The CTB bias is accordingly equal to UTD bias plus a term capturing the more stringent fiscal consolidation resulting from forcing the debt-to-GDP ratio to decline along with nominal income growth.

Equations (7) and (8) include infinite integrals that we can solve by imposing the exponential adjustment assumption employed in Frederiksen (2001b). We thus decompose implicit debt into the component related to the period up until time T

$$(9) \qquad \int_{s=t}^{T} \Delta p_s e^{-(i-\gamma)(s-t)} ds = \left[\frac{1-e^{-(i-\gamma)(T-t)}}{i-\gamma} - \frac{1-e^{-(i-\gamma+\lambda)(T-t)}}{i-\gamma+\lambda} \right] \Delta\bar{p}$$

where $\Delta\bar{p}$ is the long-term deterioration of the primary surplus and λ denotes the exponential speed of adjustment. Similarly, we find for the period after T

$$(10) \qquad \int_{s=T}^{\infty} \Delta p_s e^{-(i-\gamma)(s-t)} ds = \left[\frac{e^{-(i-\gamma)(T-t)}}{i-\gamma} - \frac{e^{-(i-\gamma+\lambda)(T-t)}}{i-\gamma+\lambda} \right] \Delta\bar{p}$$

Inserting (9) and (10) in equations (7) and (8), and simplifying the resulting expressions, yields

$$(11) \qquad p_t^S - p_t^{UTD} = \frac{e^{-(i-\gamma)(T-t)} - e^{-(i-\gamma+\lambda)(T-t)}}{1-e^{-(i-\gamma)(T-t)}} \frac{i-\gamma}{i-\gamma+\lambda}\Delta\overline{p}$$

and

$$(12) \qquad p_t^S - p_t^{CTB} = \underbrace{\left\{\frac{e^{-i(T-t)} - e^{-(i-\gamma)(T-t)}}{1-e^{-(i-\gamma)(T-t)}}\right\}}_{(-)}(i-\gamma)b_t + \underbrace{\frac{e^{-(i-\gamma)(T-t)} - e^{-(i-\gamma+\lambda)(T-t)}}{1-e^{-(i-\gamma)(T-t)}}}_{+} \frac{i-\gamma}{i-\gamma+\lambda}\Delta\overline{p}$$

Equations (11) and (12) may be used to compute the bias implied by the UTD and CTB approaches by simply inserting appropriate values for initial government debt, the long-term budget impact of ageing etc., and the nominal rates of interest and income growth, as well as the speed of adjustment of the primary surplus.

Below we do this for 19 OECD countries using the assumptions and dataset of Frederiksen (2001b), but before proceeding it is instructive to consider some general properties of the approximation error and how they relate to the conceptual differences between the three approaches.

First, the UTD bias is always positive as long as the primary balance deteriorates in the long run. That is, the primary surplus required to satisfy the condition that terminal debt is equal to initial debt will lead to an underestimation of fiscal sustainability.

Perhaps somewhat surprisingly, the UTD bias is independent of initial government debt despite the fact that the terminal constraint applies directly to the stock of formal government debt obligations. In this sense, the treatment of explicit government debt is equivalent under the S and UTD approaches. The reason is related to the constancy of total government debt under a sustainable fiscal policy pointed out above. Hence, absent autonomous changes in the primary surplus, the S approach will dictate constant government debt and, hence, a constant primary surplus. The debt path *implied* by the S method is accordingly identical to the path that is *imposed* when the UTD method is used. Initial explicit debt therefore does not give rise to any bias in this case.

The CTB bias is equal to UTD bias (because the horizon, and hence the way future changes in the primary surplus are dealt with, is truncated in the same way) plus a term representing overestimation of the burden of initial government debt. Accordingly, depending on the composition of total government debt, the close-to-balance rule may either over- or underestimate the required rate of fiscal consolidation. One important problem with the CTB approach is then that it dictates excessive fiscal consolidation for countries with high formal debt initially, but little implicit debt.

Available long-term projections of the fiscal impact of ageing typically cover the period until 2050. Assuming that the nominal interest rate is 6 per cent, that aggregate nominal income grows at a rate of 4 per cent, and that the annual speed of adjustment of the primary surplus equals 6 per cent, equations (11) and (12) then imply that the bias attributable to implicit debt amounts to 0,14 per cent of GDP for each 1 percentage point of GDP long-term decline in net tax receipts.

Accordingly, for a country facing a 5 per cent of GDP steady-state ageing burden, the required rate of fiscal consolidation is thus underestimated by 0,7 per cent of GDP when the UTD approach is used.

The bias due to initial government debt arising under the CTB approach equals 0,01 multiplied by initial government financial net debt. Using the CTB approach, the required rate of fiscal consolidation is therefore biased upwards by ½ per cent of GDP when initial government debt equals 50 percent.

Table 1 below shows the two biases for 19 OECD countries. As already noted, truncating the horizon implies that government implicit debt related to the period after 2050 is ignored. As the first three columns reveal, this effectively amounts to underestimating government implicit debt by about one-half. For the average OECD economy, this translates into an underestimation of the required rate of fiscal consolidation equal to 0,69 per cent of GDP when the UTD approach is used, but in a number of cases (Canada, Finland, Greece, Norway and Spain) the bias exceeds 1 per cent of GDP.

Table 1. Bias Under Terminal Debt Constraint in 2050 Relative to Fiscal Sustainability. Per Cent of GDP in 2002[1)]

	Government debt					Bias due to implicit debt			Bias due to explicit debt	**Total bias**	
	Implicit debt			Explicit debt	Total debt					**UTD**[2)] **rule**	**CTB**[3)] **rule**
	t to *T*	> *T*	Total			*t* to *T*	> *T*	Total			
Australia	114	107	220	5	225	-1,32	2,14	0,81	-0,05	**0,81**	**0,76**
Austria	102	96	199	50	249	-1,19	1,92	0,73	-0,51	**0,73**	**0,23**
Belgium	107	100	207	98	305	-1,24	2,01	0,76	-0,99	**0,76**	**-0,22**
Canada	194	182	376	44	419	-2,25	3,64	1,39	-0,44	**1,39**	**0,95**
Denmark	80	75	155	23	178	-0,93	1,50	0,57	-0,23	**0,57**	**0,34**
Finland	146	137	283	-42	241	-1,70	2,74	1,04	0,42	**1,04**	**1,46**
France	99	93	193	38	231	-1,16	1,87	0,71	-0,38	**0,71**	**0,33**
Germany	81	76	157	44	201	-0,94	1,52	0,58	-0,45	**0,58**	**0,13**
Greece	252	237	490	107	597	-2,94	4,74	1,80	-1,08	**1,80**	**0,73**
Ireland	134	126	260	36	297	-1,56	2,52	0,96	-0,37	**0,96**	**0,59**
Italy	27	26	53	97	150	-0,32	0,51	0,19	-0,98	**0,19**	**-0,79**
Japan	40	38	78	58	136	-0,47	0,75	0,29	-0,59	**0,29**	**-0,30**
Netherlands	131	123	253	42	295	-1,52	2,46	0,93	-0,42	**0,93**	**0,52**
Norway	463	435	898	-73	824	-5,39	8,70	3,31	0,74	**3,31**	**4,05**
Portugal	92	86	178	55	233	-1,07	1,72	0,65	-0,56	**0,65**	**0,10**
Spain	161	151	312	41	354	-1,87	3,03	1,15	-0,42	**1,15**	**0,73**
Sweden	94	88	183	-1	182	-1,10	1,77	0,67	0,01	**0,67**	**0,68**
United Kingdom	39	36	75	29	104	-0,45	0,73	0,28	-0,29	**0,28**	**-0,02**
United States	115	108	223	43	266	-1,34	2,16	0,82	-0,43	**0,82**	**0,39**
Unwtd. average	130	122	252	37	289	-1,51	2,44	0,93	-0,37	**0,93**	**0,56**
GPD-wtd. avg.	96	90	187	46	233	-1,12	1,81	0,69	-0,47	**0,69**	**0,22**
GPD-wtd. EU avg.	84	79	163	49	213	-0,98	1,58	0,60	-0,50	**0,60**	**0,10**
GPD-wtd. EMU avg.	93	87	179	55	235	-1,08	1,74	0,66	-0,56	**0,66**	**0,10**

Notes:

1) The results are based on the same basic assumptions and sources as Frederiksen (2001b) except for initial government net debt which is from OECD Economic Outlook no. 72, December 2002. The nominal rates of interest and growth are 6 and 4 per cent, respectively, while the exponential adjustment speed used to compute implicit debt obligations is 6 per cent.

2) Unchanged Terminal Debt, i.e. government financial net debt in 2050 is constrained to be equal to government financial net debt in 2002.

3) Close To Balance; i.e. the ratio of government financial net debt to GDP is constrained to decline with nominal income growth from 2002 to 2050.

As equation (6) demonstrates, the sustainable primary surplus is equal to the growth-adjusted interest rate times total government debt. With the former equal to 2 per cent and the latter at 2,3 times GDP on average for the 19 countries covered in the table, fiscal sustainability necessitates a primary surplus of 4,7 per cent of GDP in the initial year. Using the assumption of unchanged terminal debt then leads to an underestimation of the required primary surplus by about one-seventh.

The CTB method implies lower bias on average because the two sources of error tend to offset each other. For the average OECD country, the CTB bias is about 0,2 per cent of GDP. Hence, while the CTB approach also tends to underestimate the fiscal challenges faced by OECD governments, the magnitude is somewhat smaller.

The table also illustrates how the CTB bias may be either positive or negative depending on the composition of total government debt. For example, the Dutch and Belgian governments face roughly identical total debt burdens of approximately 3 times GDP. However, explicit debt is much higher in Belgium, whereas the ageing burden is more severe in the Netherlands. Applying the CTB method then leads to an *under*estimation of the required primary surplus in the Netherlands by about 0,5 per cent of GDP, while for Belgium it is *over*estimated by 0,2 per cent GDP.

One peculiar aspect of the CTB approach is that countries with positive initial net financial assets are effectively assumed to liquidate those assets, in turn implying an additional source of underestimation of the required rate of fiscal consolidation. This is the case for Finland, Norway and Sweden. Thus, for Finland, the shortfall under the CTB method amounts to 1½ per cent of GDP compared to a policy of "true" fiscal sustainability.

These examples thus indicate that the two analytical short-cuts involving truncation of the time horizon tend to complicate the cross-country comparison of fiscal stance.

As mentioned previously, the bias attributable to truncation declines when the horizon is extended. *Table 2* shows the consequences of varying the terminal year between 2025 and 2200.

Table 2. Time Horizon and Average Bias Under Terminal Debt Constraint Rules. Per Cent of GDP in 2002

Bias:	Horizon (terminal year of UTD and CTB rules)				
	25	*50*	*100*	*150*	*200*
Implicit debt	1,49	0,69	0,19	0,07	0,02
Explicit debt	-0,91	-0,47	-0,14	-0,05	-0,02
UTD rule	**1,49**	**0,69**	**0,19**	**0,07**	**0,02**
CTB rule (mean)	**0,58**	**0,22**	**0,05**	**0,02**	**0,01**
CTB rule (mean absolute)	**0,93**	**0,42**	**0,12**	**0,04**	**0,01**

Also shown is the average absolute bias under the CTB rule. Comparison of the mean and mean absolute bias reveals that about half of the reduction in average bias relative to the UTD approach is due to the fact that the sign of the CTB bias may be either positive or negative. The average absolute error when the CTB method is used thus equals 0,4 per cent of GDP at a 50-year horizon.

The table also shows that a *very* long horizon must be employed in order for the bias to assume insignificant proportions. In other words, the UTD and CTB approaches provide a reliable assessment of fiscal sustainability only at horizons of 150 years or more. This outcome is the direct result of the fact that, as already noted, a very substantial portion of government implicit debt is related to the period after 2050.

2. Conclusions

This paper provides an assessment of the conceptual and quantitative differences between three alternative ways of deriving long-term indicators of fiscal stance. The conceptually most appealing one is fiscal sustainability, where an infinite horizon is maintained and the sustainability constraint is imposed directly (and, in contrast to the use of essentially arbitrary terminal values for government debt, *only*) on fiscal instruments.

We use that approach as a benchmark and compare it with two alternative ones where the time horizon is truncated, thereby implying that terminal values for government debt in finite time are needed in order to render the quantitative assessment of fiscal sustainability computationally feasible.

Within the typical time frame of long-term fiscal projections, i.e. about 50 years, requiring terminal government debt to equal initial debt is shown to underestimate the required primary surplus in the OECD countries by about 0,7 per cent of GDP on average. And the time horizon of the projections would have to be extended significantly – to more than 150 years – in order to reduce the bias to a trivial magnitude.

An alternative, finite horizon method is based on the constraint that terminal government debt equals initial debt diluted by nominal income growth. Given this requirement average bias is reduced to about 0,2 per cent of GDP. However, cross-country comparability is hampered by the fact that the composition of total government debt (i.e., the split between explicit and implicit liabilities) affects both the sign and the magnitude of the approximation error.

We may therefore conclude that, for the OECD countries, the two finite-horizon alternatives tend to imply too little fiscal consolidation compared to what is required for "true" fiscal sustainability. That is, as a by-product, the (technically motivated) imposition of a fixed horizon typically leads to an xcessively optimistic picture of current fiscal stance.

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